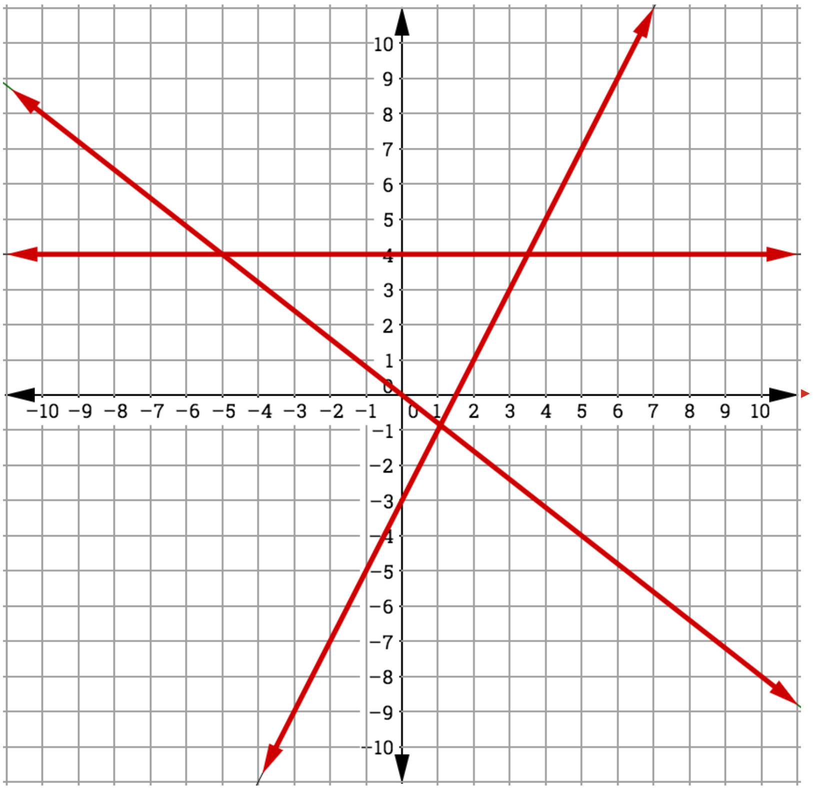
Section 8: Summary of Functions

Section 8 – Topic 1
Comparing Linear, Quadratic, and Exponential Functions – Part 1

Complete the table below to describe the characteristics of linear functions.

Linear Functions	
Equation	$y = mx + b$
Shape	**linear**
Rate of Change	**constant**
Number of x-intercepts	**0 or 1 or infinitely many**
Number of y-intercepts	1
Number of vertices	0
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \in \mathbb{R}\}$ **or** $\{y \mid y = some\ constant\}$

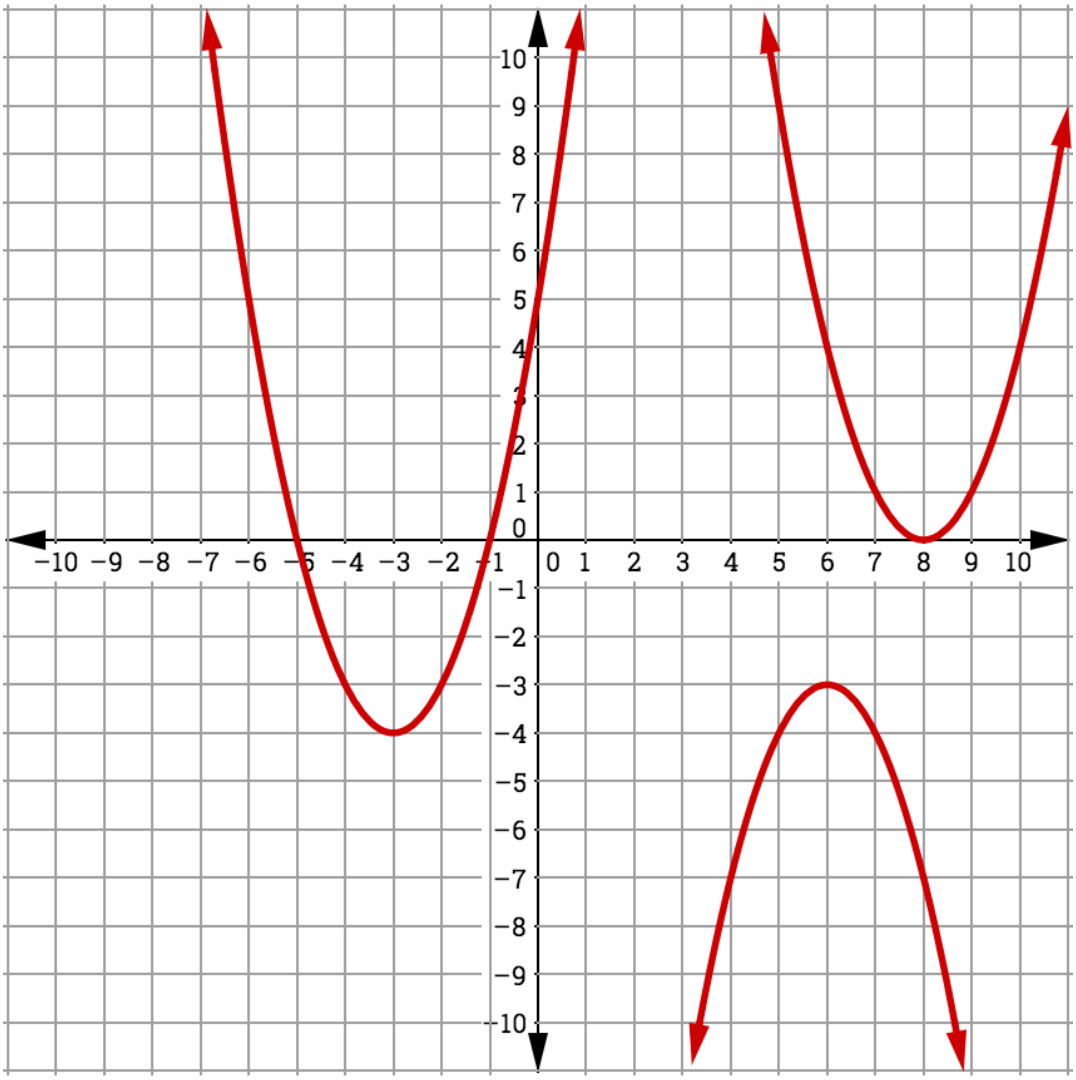
Sketch the graphs of three linear functions that show all the possible combinations above.



Complete the table below to describe the characteristics of quadratic functions.

Quadratic Functions	
Equation	$y = ax^2 + bx + c$, **where** $a \neq 0$
Shape	**parabola**
Rate of Change	**Not constant**
Number of x-intercepts	**0, 1, or 2**
Number of y-intercepts	**1**
Number of vertices	**1**
Domain	$\{x \mid x = \mathbb{R}\}$
Range	$\{y \mid y \geq some\ constant\}$ **or** $\{y \mid y \leq some\ constant\}$

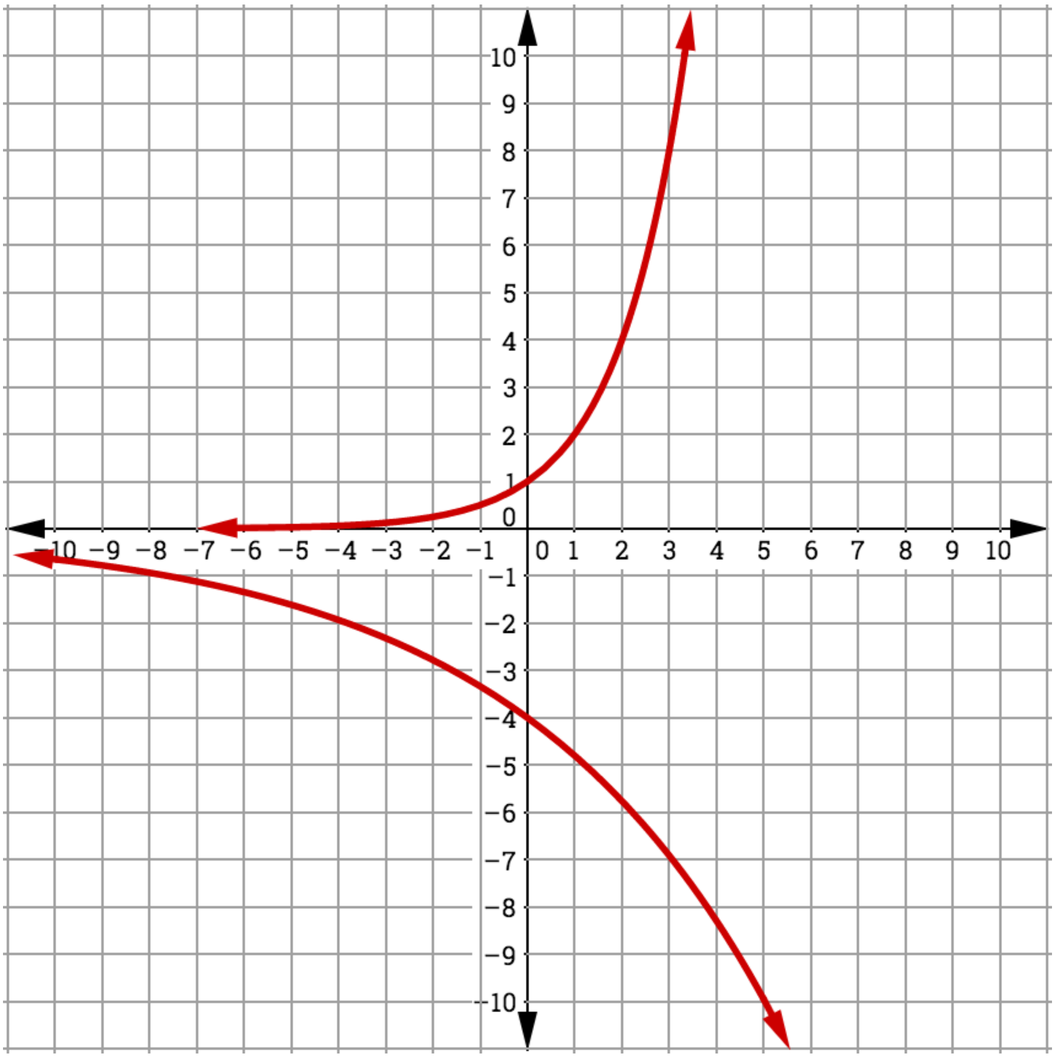
Sketch the graphs of three quadratic functions that show all the possible combinations above.



Complete the table below to describe the characteristics of exponential functions.

Exponential Functions	
Equation	$y = a \cdot b^x$ **where** $a \neq 0$ **and** $b > 0.$
Shape	**exponential**
Rate of Change	**Not constant**
Number of x-intercepts	**0 or 1**
Number of y-intercepts	**1**
Number of vertices	**0**
Domain	$\{x \mid x = \mathbb{R}\}$
Range	$\{y \mid y > some\ constant\}$ **or** $\{y \mid y < some\ constant\}$

Sketch the graphs of two exponential functions that show all the possible combinations above.



Consider the following tables that represent a linear and a quadratic function and find the differences.

Linear Function	
x	$f(x)$
0	5
1	7
2	9
3	11
4	13

2

2

2

2

Quadratic Function	
x	$f(x)$
0	3
1	4
2	7
3	12
4	19

1

3

5

7

2

2

2

How can you distinguish a linear function from a quadratic function?

The first differences in a linear function are constant. The second differences, but not the first of a quadratic function are constant.

Consider the following table that represents an exponential function.

Exponential Function	
x	$f(x)$
0	1
1	3
2	9
3	27
4	81
5	243

$\times 3$

$\times 3$

$\times 3$

$\times 3$

$\times 3$

How can you determine if a function is exponential by looking at a table?

There is a common ratio.

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Section 8 – Topic 2

Comparing Linear, Quadratic, and Exponential Functions – Part 2

Let's Practice!

1. Identify whether the following key features indicate a model could be linear, quadratic, or exponential.

Key Feature	Linear	Quadratic	Exponential
Rate of change is constant.	●	○	○
2^{nd} differences, but not 1^{st}, are constant.	○	●	○
Graph has a vertex.	○	●	○
Graph has no x-intercept.	●	●	●
Graph has two x-intercepts.	○	●	○
Graph has one y-intercept.	●	●	●
Domain is all real numbers.	●	●	●
Range is $\{y\|y > 0\}$.	○	○	●
Range is $\{y\|y \leq 0\}$.	○	●	○
Range is all real numbers.	●	○	○

Try It!

2. Determine whether each table represents a linear, quadratic, or exponential function.

x	y
0	1
1	2
2	5
3	10
4	17

x	y
0	7
3	13
6	19
9	25
15	37

x	y
0	2
1	6
2	18
3	54
4	162

○ Linear
[✓ ○ Quadratic]
○ Exponential

[✓ ○ Linear]
○ Quadratic
○ Exponential

○ Linear
○ Quadratic
[✓ ○ Exponential]

BEAT THE TEST!

1. Identify whether the following real-world examples should be modeled by a linear, quadratic, or exponential function.

Real-World Example	**Linear**	**Quadratic**	**Exponential**
Growing a culture of bacteria	○	○	●
The distance a Boeing 737 MAX can travel at a certain speed over a given period of time	●	○	○
Kicking a ball into the air	○	●	○
Running a race at a constant speed	●	○	○
A pumpkin decaying	○	○	●
Jumping from a high dive	○	●	○

2. Complete the following table so that $f(x)$ represents a linear function and $g(x)$ represents an exponential function.

Answers Vary. Sample answers:

x	$f(x)$	$g(x)$
-5	-8	1
-4	2	3
-3	12	9
-2	22	27
-1	32	81

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Section 8 – Topic 3
Comparing Arithmetic and Geometric Sequences

The founder of a popular social media website is trying to inspire gifted algebra students to study computer programming. He is offering two different incentive programs for students.

Option 1: Students will earn one penny for completing their first math, science, or computer-related college course. The amount earned will double for each additional course they complete.

Option 2: Students will earn one penny for completing their first math, science, or computer-related college course. For each subsequent course completed, they will earn $\$100.00$ more than the previous course.

Write an explicit formula for each option.

Option 1: $a_n = 0.01 \cdot 2^{n-1}$

Option 2: $a_n = 0.01 + 100(n-1)$

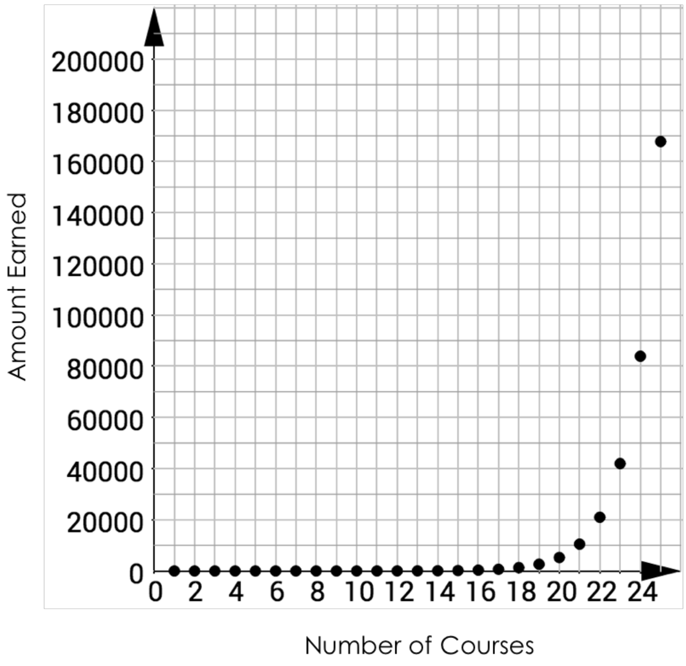
Compare the two scholarship options in the tables below.

Option 1	
Course	**Amount**
1	$0.01
2	$0.02
3	$0.04
4	$0.08
5	$0.16
6	$0.32
7	$0.64
8	$1.28
9	$2.56
10	$5.12
11	$10.24
12	$20.48
13	$40.96
14	$81.92
15	$163.84
16	$327.68
17	$655.36
18	$1,310.72
19	$2,621.44
20	$5,242.88
21	$10,485.76
22	$20,971.52
23	$41,943.04
24	$83,886.08
25	$167,772.16

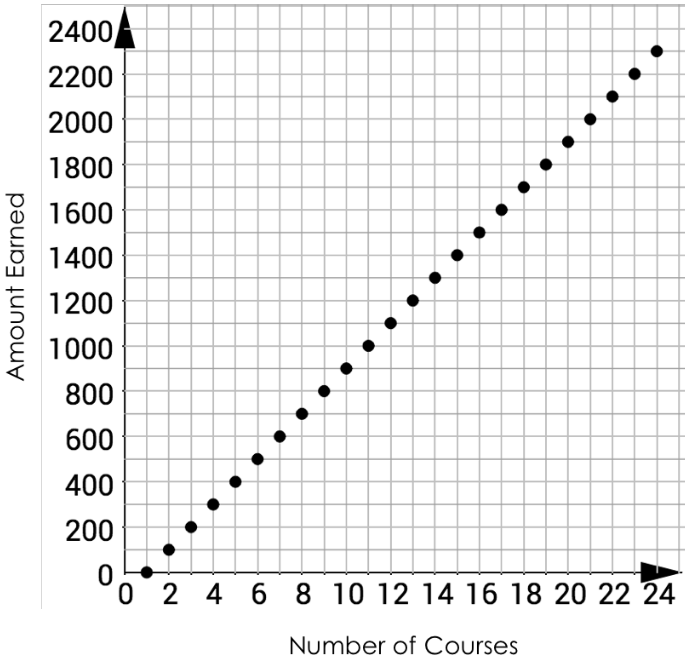
Option 2	
Course	**Amount**
1	$0.01
2	$100.01
3	$200.01
4	$300.01
5	$400.01
6	$500.01
7	$600.01
8	$700.01
9	$800.01
10	$900.01
11	$1,000.01
12	$1,100.01
13	$1,200.01
14	$1,300.01
15	$1,400.01
16	$1,500.01
17	$1,600.01
18	$1,700.01
19	$1,800.01
20	$1,900.01
21	$2,000.01
22	$2,100.01
23	$2,200.01
24	$2,300.01
25	$2,400.01

Compare the two scholarship options in the graphs below.

Option 1



Option 2



Option 1 is a geometric sequence.

- Consecutive terms in this sequence have a common __**ratio**___.
- This geometric sequence follows a(n) _____**Exponential**_____ pattern.
- Evaluate the domain of this function.
  The set of Natural numbers: $\{x|x = 1, 2, 3, 4 \ldots\}$

Option 2 is an arithmetic sequence.

- Consecutive terms in this sequence have a common __**difference**______.
- Arithmetic sequences follow a(n) _____**Linear**_____________ pattern.
- Evaluate the domain of this function.
  The set of Natural numbers: $\{x|x = 1, 2, 3, 4 \ldots\}$

What can be said about the domain of arithmetic and geometric sequences?

The domain is a subset of the integers.

Let's Practice!

1. Consider the two scholarship options for studying computer science.

   a. Which scholarship option is better if your college degree requires 10 math, engineering, or programming courses?

      Option 2

   b. What if your degree requires 25 math, engineering, or programming courses?

      Option 1

   c. Do you think that these graphs represent discrete or continuous functions? Justify your answer.

      Discrete, because you cannot take half of a course.

   d. Do you think Option 1 would ever be offered as a scholarship? Why or why not?

      Answers vary: Probably not unless there was a cap on the number of courses you could take.

Try It!

2. Pablo and Lily are saving money for their senior trip next month. Pablo's goal is to save one penny on the first day of the month and to triple the amount he saves each day for one month. Lily's goal is to save $10.00 on the first day of the month and increase the amount she saves by $5.00 each day.

   a. Pablo's savings plan is an example of a(n)

      ○ arithmetic sequence.
      ● geometric sequence.

   b. Lily's savings plan is an example of a(n)

      ● arithmetic sequence.
      ○ geometric sequence.

   c. Which person do you think will be able to meet his/her goal? Explain.
      Answers vary. Sample answer: Lily, Pablo's amount to save would get exponentially large.

3. Circle the best answers to complete the following statement.

   Arithmetic sequences follow a(n) [✓ linear] | exponential | quadratic pattern, whereas geometric sequences follow a(n) linear | [✓ exponential] | quadratic pattern, and the domain of both sequences is a subset of the [✓ integers] | radicals | exponents.

BEAT THE TEST!

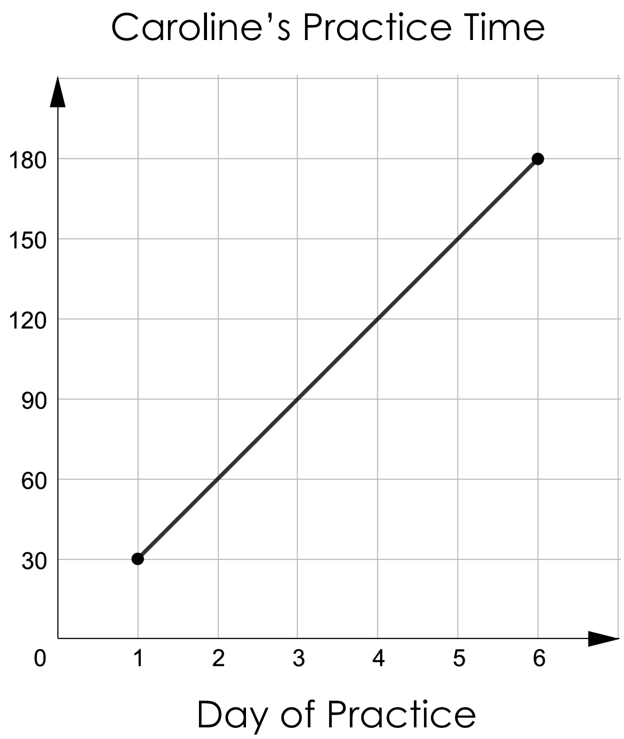
1. Caroline and Chris have a piano recital in six days. They are discussing their plans to increase the frequency of practice. Caroline's practice plans are listed in the table below. Chris plans to practice half of Caroline's time on Day 1, but will double his practice time every day for the remaining five days.

Part A: List Caroline's and Chris's practice times on the tables below.

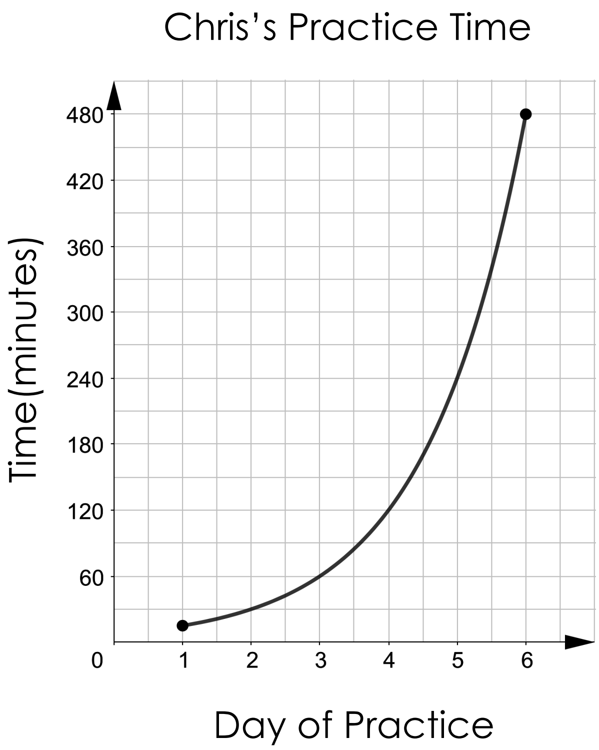
Caroline's Practice Time (in Minutes)	
Day 1	30
Day 2	60
Day 3	90
Day 4	120
Day 5	150
Day 6	180

Chris's Practice Time (in Minutes)	
Sunday	**15**
Monday	**30**
Tuesday	**60**
Wednesday	**120**
Thursday	**240**
Friday	**480**

Part B: Compare the graphs of Caroline's and Chris's practice times. Identify each graph as linear or exponential.



Linear



Exponential

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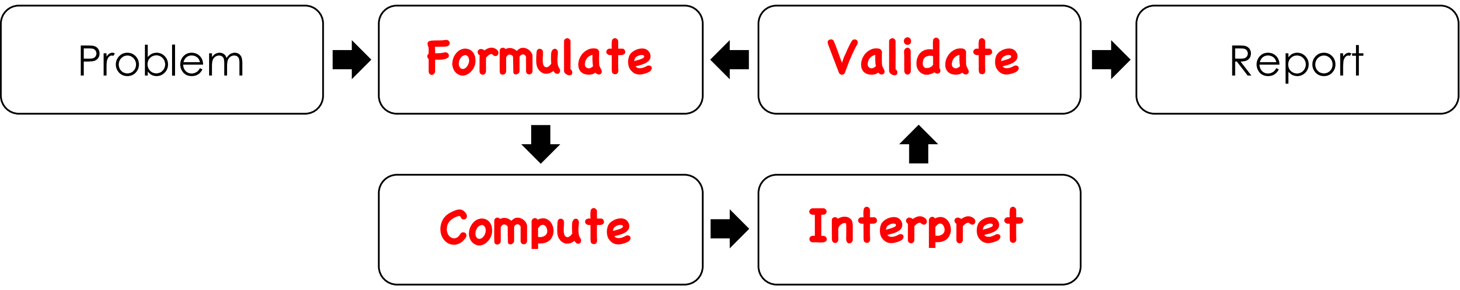
Section 8 – Topic 4
Modeling with Functions

Let's discuss the modeling cycle process.

Consider and complete the following diagram that displays the modeling cycle process.



Let's Practice!

1. The table below represents the population estimates (in thousands) of the Cape Coral-Fort Myers metro area in years since 2010. Employ the modeling cycle to create a graph and a function to model the population growth. Use the function to predict the population in 2020.

x	0	1	2	3	4	5	6
$f(x)$	619	631	645	661	679	699	721

Problem – Identify the variables in the situation and select those that represent essential features.

a. What are the variables in this situation and what do they represent?

$x =$ years since 2010
$f(x) =$ population estimates in thousands

Formulate a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables.

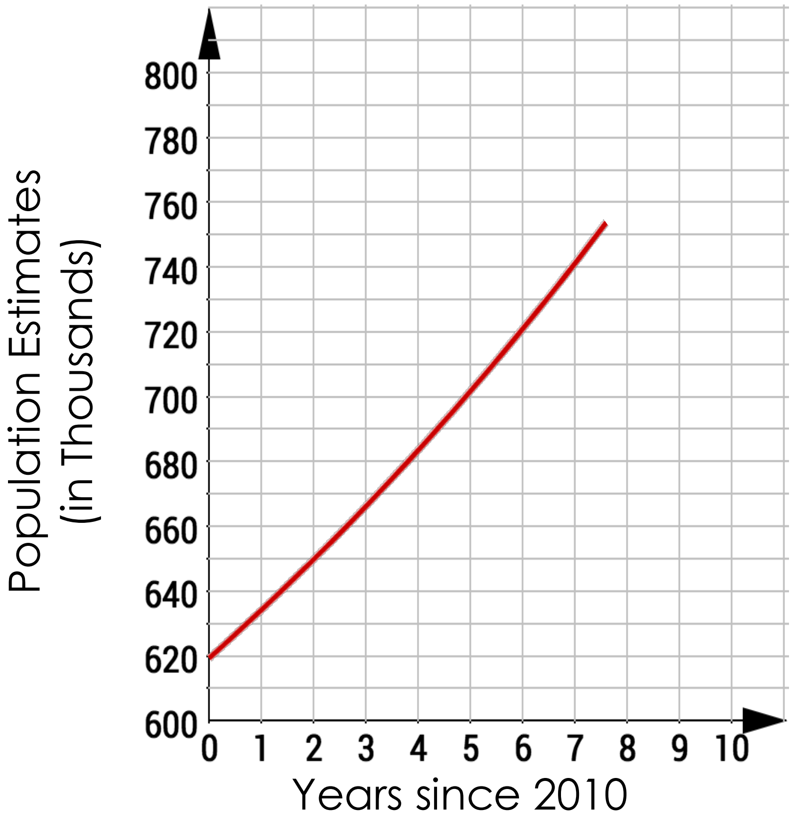
b. Determine what type of function models the context.

Quadratic function

Compute – Analyze and perform operations on these relationships to draw conclusions.

c. Sketch the graph and find the function that models the table.

Population of Cape Coral-Fort Myers Metro Area



$f(x) = x^2 + 11x + 619$

d. Use the model to predict the population in the year 2020.

$\mathbf{829,000}$

Interpret the results of the mathematics in terms of the original situation.

e. What do the results tell you about the population growth in Cape Coral-Fort Myers metro area as it relates to the original table?

Growing at a relatively fast rate

Validate the conclusions by comparing them with the situation, and then either improve the model, or, if it is acceptable, move to the reporting phase.

f. What methods can we use to validate the conclusions?

$\boldsymbol{f(3) = 661}$

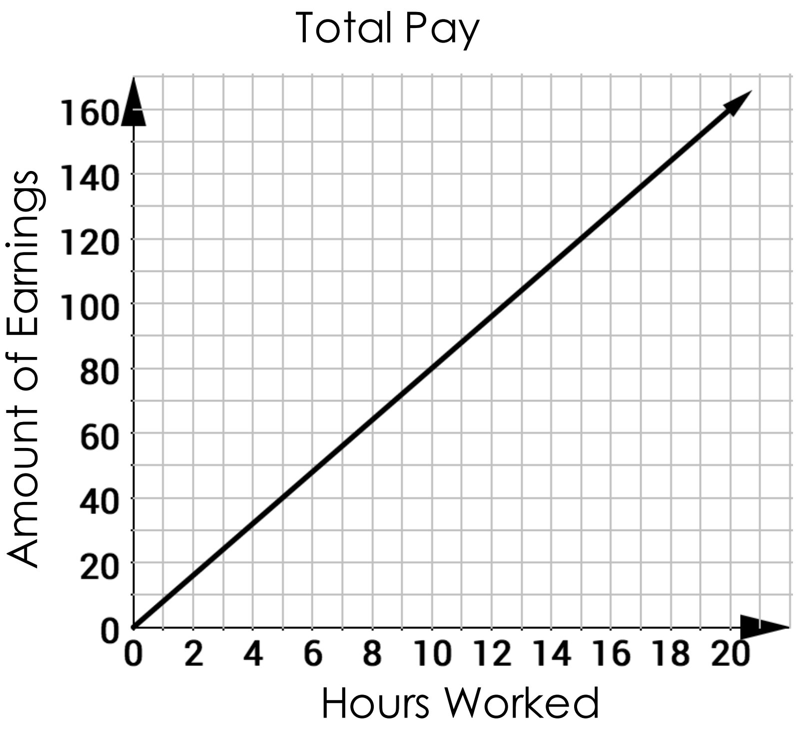
Report on the conclusions and the reasoning behind them.

g. What key elements should be included in your report?

Function that we found to model, population estimate for 2020, validation and interpretation

Try It!

2. According to Florida's Child Labor Law, minors who are 14 or 15 years old may work a maximum of 15 hours per week, and minors that are 16 or 17 years old may work a maximum of 30 hours per week. The relationship between the number of hours that a 15-year old minor in Florida works and his total pay is modeled by the graph below. What is the maximum amount that he can earn in a week?



Phase 1: **Problem**

a. Identify the variables in the situation and what they represent.
$x =$ **hours worked**
$f(x) =$ **amount earned in dollars**

Phase 2: **Formulate**

b. What type of function can be represented by this graph?
Linear function

c. Describe the end behavior of the graph.
As x increases, $f(x)$ increases

d. What does the end behavior tell you about the function?
The more hours he works the more earnings he make

Phase 3: **Compute**

e. What strategy will you use to create the model for this situation?
$\boldsymbol{f(x) = mx + b}$, **find** $\boldsymbol{m}$ **and** $\boldsymbol{b}$

f. Find the function of the graph.
$\boldsymbol{f(x) = 8x}$

Phase 4: **Interpret**

g. Complete the following statement.

The domain that best describes this situation is

$\{x | x \epsilon$
- ● rational numbers}.
- ○ natural numbers}.
- ○ whole numbers}.

h. What constraints on the domain would exist for a 14-year old? A 17-year old?
14: $\boldsymbol{\{x | 0 \leq x \leq 15\}}$
17: $\boldsymbol{\{x | 0 \leq x \leq 30\}}$

i. How much does the student make per hour? Justify your answer algebraically.
$8, this is the slope of the function

Phase 5: **Validate**

j. Verify that your function accurately models the graph.

$f(15) = 120$

k. Are there other ways to validate your function?

Choose other points

Phase 6: **Report**

l. What would you report?

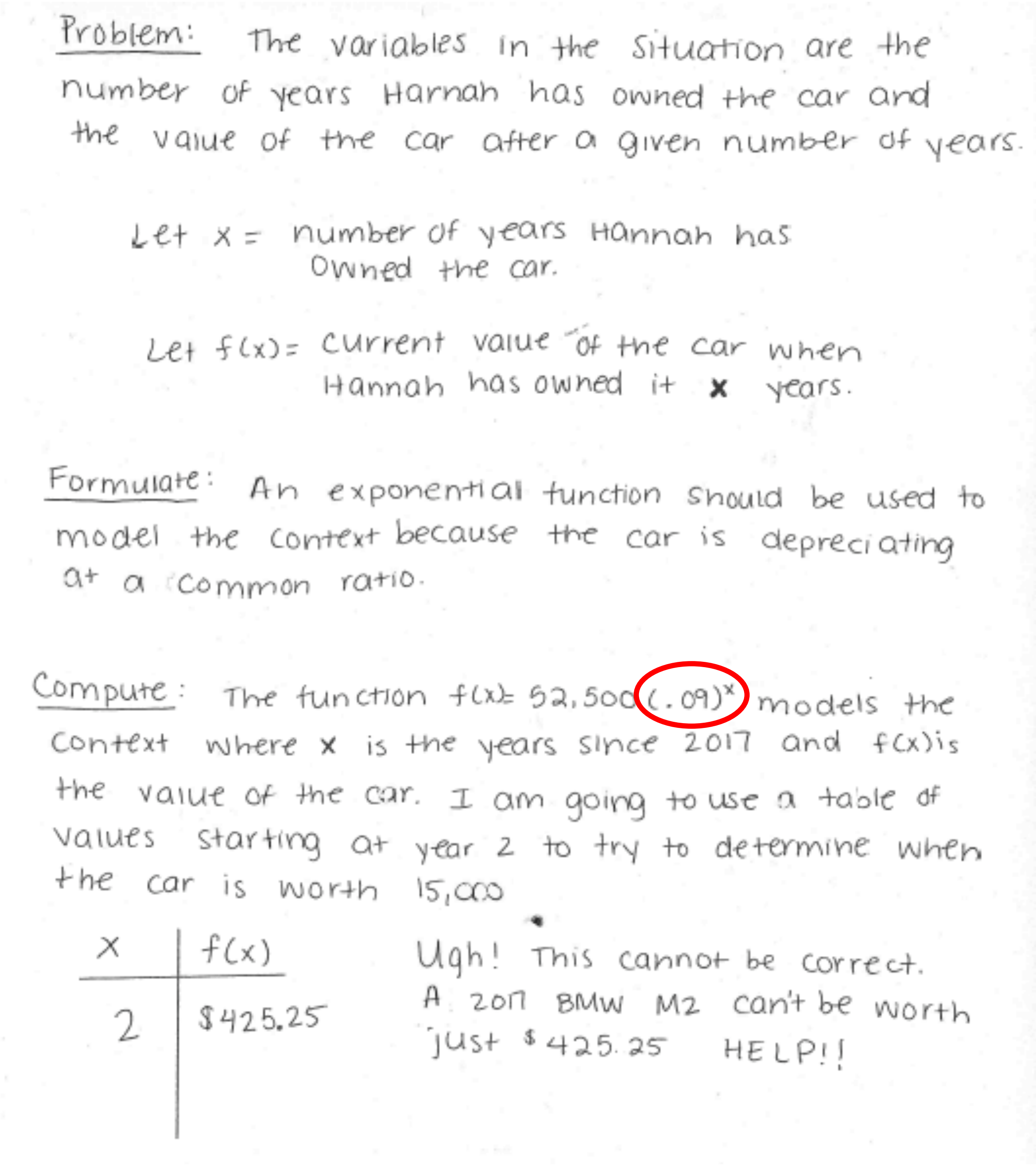
The equation that models the function is linear. $f(x) = 8x$ was determined to be the relationship where $x =$ hours worked and $f(x) =$ amount earned in dollars. For a 15-year-old, they can work a maximum of 15 hours per week. Thus, $f(15) = \$120$ is the maximum amount to earn in a week.

BEAT THE TEST!

1. Dariel employed the modeling cycle to solve the following problem.

> Hannah’s uncle works at the BMW plant in Spartanburg, South Carolina. He purchased a 2017 BMW M2 for Hannah at the manufacturer’s suggested retail price (MSRP) of $52,500. Suppose over the next ten years, the car will depreciate an average of 9% per year. Hannah wishes to sell the car when it is valued at $22,000. When should she sell the car?

When Dariel got to the compute phase, he knew something was wrong. His work is shown below.



Problem: The variables in the situation are the number of years Harnah has owned the car and the value of the car after a given number of years.

Let x = number of years Hannah has Owned the car.

Let f(x) = current value of the car when Hannah has owned it x years.

Formulate: An exponential function should be used to model the context because the car is depreciating at a common ratio.

Compute: The function $f(x) = 52{,}500(.09)^x$ models the context where x is the years since 2017 and f(x) is the value of the car. I am going to use a table of values starting at year 2 to try to determine when the car is worth 15,000

x	f(x)
2	$425.25

Ugh! This cannot be correct. A 2017 BMW M2 can't be worth just $425.25 HELP!!

Part A: Critique his reasoning and give feedback on where he went wrong.

The function should have been $f(x) = 52500(0.91)^x$ so that between year 9 and 10 the value is $\$22,466$ and $\$20,444$, respectively.

Part B: Complete the modeling cycle.

The function that models the value of the car is $f(x) = 52500(0.91)^x$, where x is the number of years Hannah has owned the car. If she wants to sell it when it is still worth $\$22,000$, she should sell shortly after 9 years of ownership.

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Section 8 – Topic 5
Understanding Piecewise-Defined Functions

What is a ***piecewise function***?

- A function made up of distinct "____pieces____" based on different rules for the ____domain____.
- The "pieces" of a piecewise function are graphed together on the same coordinate plane.
- The ***domain*** is the ____input____, or the x-values.
- The ***range*** is the _y_-values, or output.
- Since it is a function, all "pieces" pass the vertical line test.

Describe an example of a piecewise function used in our daily lives.

Traveling at different speeds over intervals of time on a road trip.

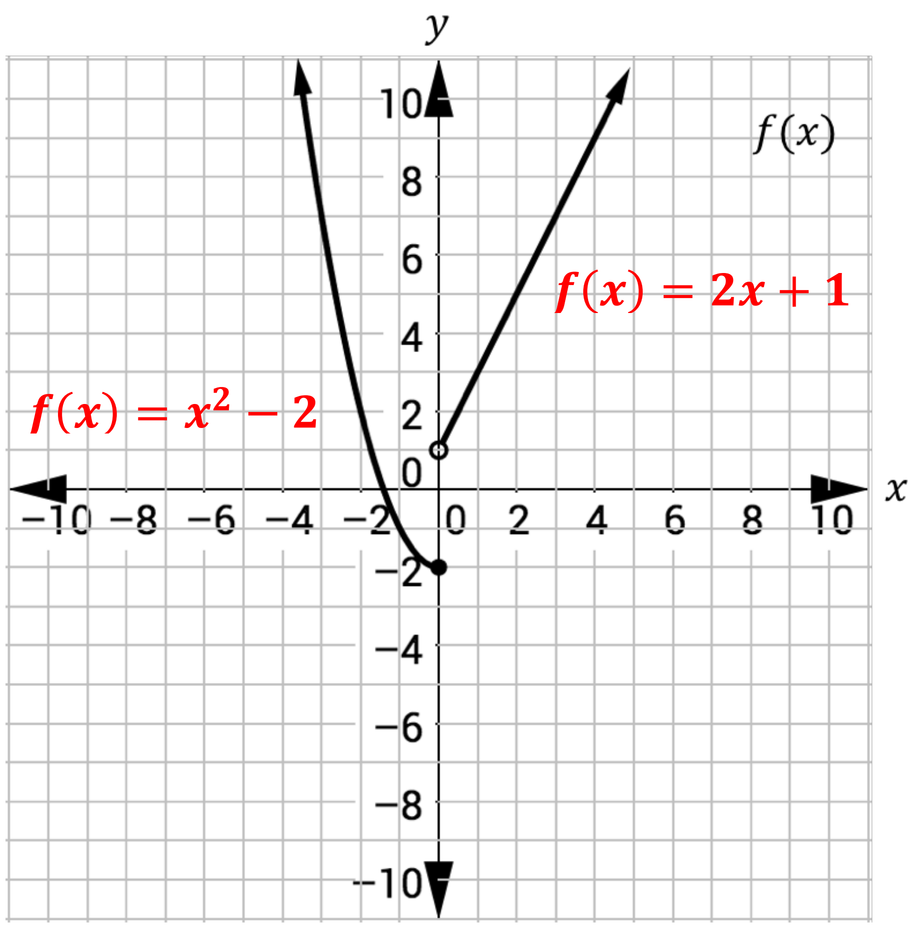
Getting paid hourly pay at minimum wage and then getting paid overtime rate.

Consider the following piecewise-defined function.

$$f(x) = \begin{cases} x^2 - 2, \text{when } x \leq 0 \\ 2x + 1, \text{when } x > 0 \end{cases}$$

- Each function has a defined domain value, or rule.
    - x is less than or equal to zero for the first function.
    - x is greater than zero for the second function.

- Both of these functions will be on the same graph. They are the "pieces" of this completed piecewise-defined function.



Label the "pieces" of $f(x)$ above.

Let’s note some of the features of the graph.

- The domain of the piecewise graph can be represented with intervals. If we define the first interval as $x \leq 0$, the second interval would be $\mathbf{x > 0}$.
- The graph is nonlinear (curved) when the domain is $\mathbf{x \leq 0}$.
- The graph is linear when the domain is $\mathbf{x > 0}$.
- There is one closed endpoint on the graph, which means that the particular domain value, zero, is **included** in that piece of the function. This illustrates the inclusion of zero in the function $\mathbf{x^2 - 2}$.
- There is one open circle on the graph, which means that the particular value, zero, is **not included** in that piece of the function. This illustrates the constraint that $x > 0$ for the function $\mathbf{2x + 1}$.

Let's Practice!

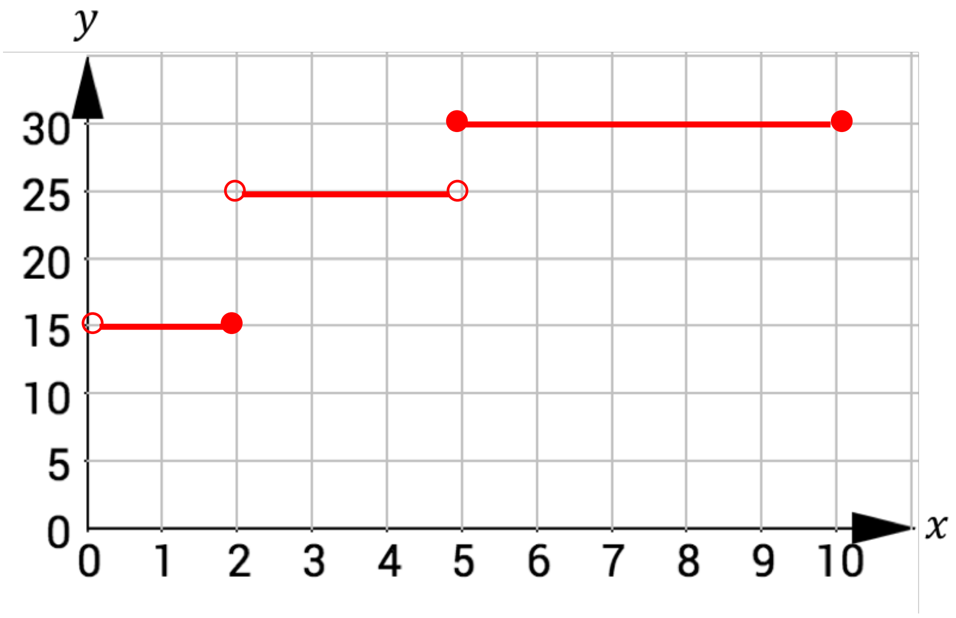
1. Airheadz, a trampoline gym, is open seven days a week for ten hours a day. Their prices are listed below:

Two hours or less: $\$15.00$
Between two and five hours: $\$25.00$
Five or more hours: $\$30.00$

The following piecewise function represents their prices:

$$f(x) = \begin{cases} 15, \text{when } 0 < x \leq 2 \\ 25, \text{when } 2 < x < 5 \\ 30, \text{when } 5 \leq x \leq 10 \end{cases}$$

Graph the above function on the following grid.



- $f(x)$ is a special type of piecewise function known as a ___**step**___ function, which resembles a series of steps.

- Step functions pair every x-value in a given interval (particular section of the ___**domain**___) with a single value in the range (__y__-value).

Try It!

2. Consider the previous graph in exercise 1.

   a. How many pieces are in the step function? Are the pieces linear or nonlinear?

   Three pieces and linear

   b. How many intervals make up the step function? What are the interval values?

   Three intervals: $(0,2], (2,5), [5,10]$

   c. Why are open circles used in some situations and closed circles in others?

   In open circles number is not included in the interval and in closed circles numbers are included in interval.

   d. How do you know this is a function?

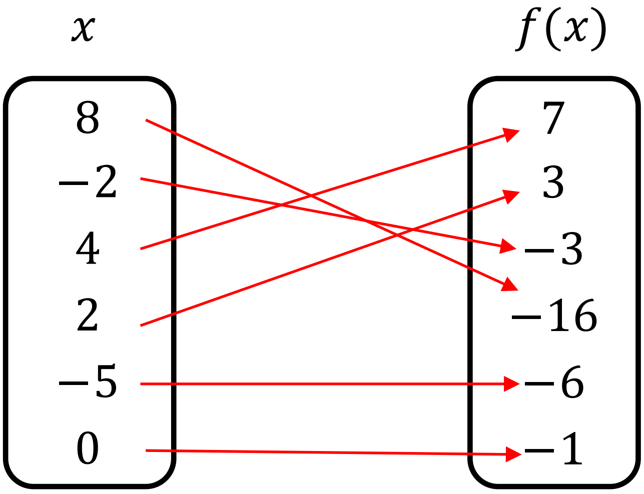
   Vertical line test, for every input there is a unique output.

   e. What is the range of this piecewise function?
   15, 25, 30

BEAT THE TEST!

1. Evaluate the piecewise-defined function for the given values of x by matching the domain values with the range values.

$$f(x)=\begin{cases} x-1, & x\leq -2 \\ 2x-1, & -2<x\leq 4 \\ -3x+8, & x>4 \end{cases}$$



Answers: $(-5,-6),(-2,-3),(0,-1),(2,3),(4,7),(8,-16)$

2. Complete the following sentences by choosing the correct answer from each box.

Part A: Piecewise-defined functions are represented by

○ one function
○ at least one function
● at least two functions

that must correspond

to

● different domain values
○ different range values
○ real numbers

Part B: When evaluating piecewise-defined functions, choose which equation to use based on the

then

○ constant
● x-value
○ slope

substitute and evaluate

using

○ exponent rules
● order of operations
○ your instincts

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Section 8 – Topic 6
Finding Zeros of Polynomial Functions of Higher Degrees

How can you find zeros when given the graph of a polynomial function?

Find the x-intercepts.

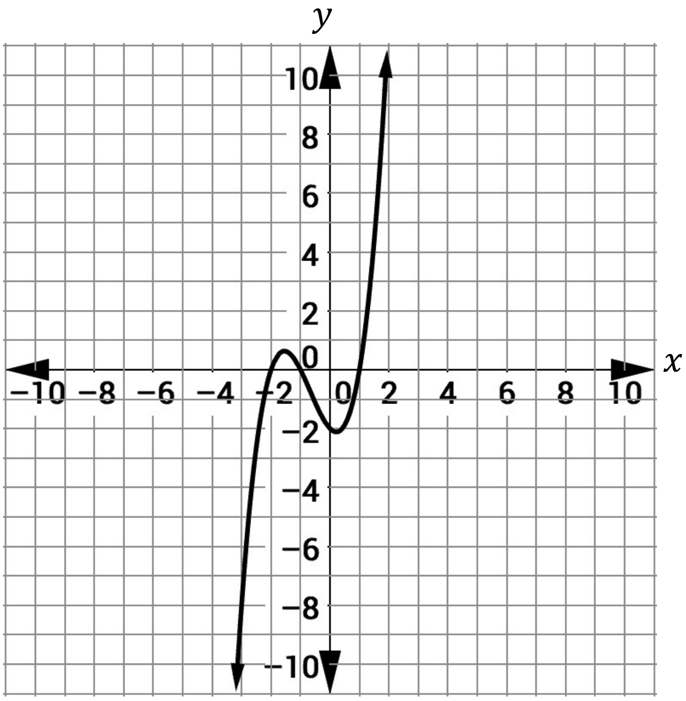
How can you find zeros when given the equation of a polynomial function in factored form?

Use the zero product property and set each factor equal to zero and solve.

How do you determine if x is a solution or zero for $f(x)$?

If x is a solution, then $f(x) = 0$.

Consider the following graph of $f(x)$.



What are the zeros of $f(x)$?

$x = -2$, $x = -1$, and $x = 1$

Consider the following fourth degree polynomial function.

$$g(x) = x^4 - 4x^2$$

Find the range of $g(x)$ for the given domain $\{-2, -1, 0, 1, 2\}$.

$g(-2) = (-2)^4 - 4(-2)^2 = 16 - 4(4) = 0$
$g(-1) = (-1)^4 - 4(-1)^2 = 1 - 4 = -3$
$g(0) = 0^4 - 4(0)^2 = 0$
$g(1) = 1^4 - 4(1)^2 = 1 - 4 = -3$
$g(2) = 2^4 - 4(2)^2 = 16 - 4(4) = 0$

Range: $\{0, -3\}$

Does the above domain contain zeros of $g(x)$? Justify your answer.
Yes, $-2, 0,$ **and** 2 **are all zeros because for those values** $g(x) = 0$.

Consider the following third degree polynomial function.

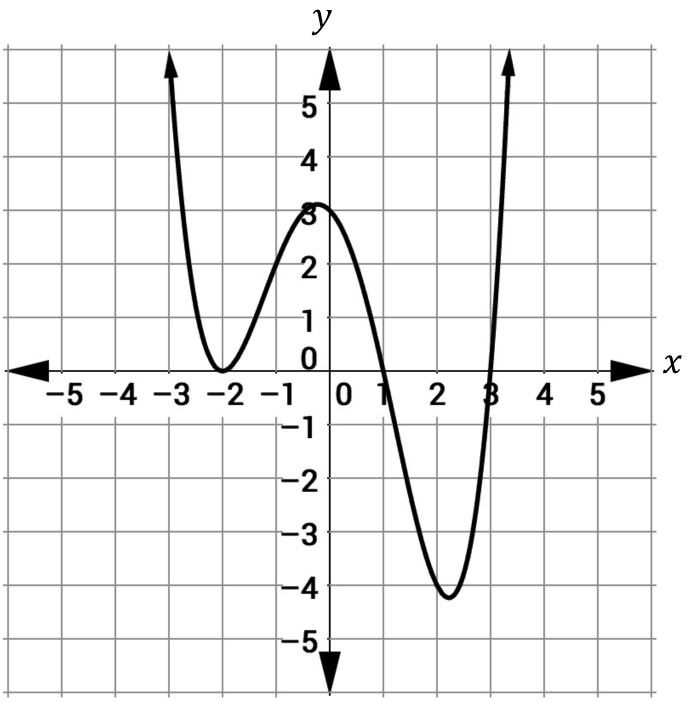
$$h(x) = -x^3 - 5x^2$$

Find the zeros of the function $h(x)$.

$-x^3 - 5x = 0$
$-x^2(x + 5) = 0$
$-x^2 = 0$ **or** $x + 5 = 0$
$x = 0$ **or** $x = -5$

Let's Practice!

1. Consider the following graph of $f(x)$.



What are the zeros of $f(x)$?

$\boldsymbol{x = -2}$, $\boldsymbol{x = 1}$, **and** $\boldsymbol{x = 3}$.

2. What are the zeros of $g(x) = x(x+1)(x-2)^2$?

$\boldsymbol{x = 0,\ x + 1 = 0,\ (x - 2)^2 = 0}$

$\boldsymbol{x = 0,\ x = -1,\ x = 2}$

Try It!

3. Consider the function $h(x) = x^3 - 3x^2 + 2$.

a. Find the range of $h(x)$ given domain $\{-1, 1, 3\}$.

$\mathbf{f(-1) = (-1)^3 - 3(-1)^2 + 2 = -1 - 3(1) + 2 = -2}$

$\mathbf{f(1) = (1)^3 - 3(1)^2 + 2 = 1 - 3(1) + 2 = 0}$

$\mathbf{f(3) = (3)^3 - 3(3)^2 + 2 = 27 - 3(9) + 2 =}$

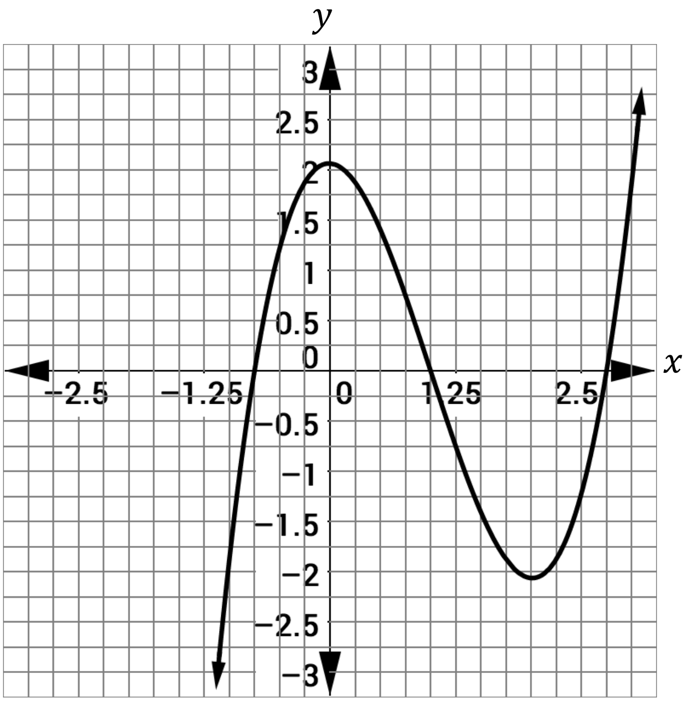
$\mathbf{27 - 27 + 2 = 2}$

$\mathbf{\{-2, 0, 2\}}$

b. Are any zeros of $h(x)$ found in the above domain? Justify your answer.

Yes, $x = 1$ is a zero because $f(1) = 0$.

c. Consider the graph of $h(x)$.



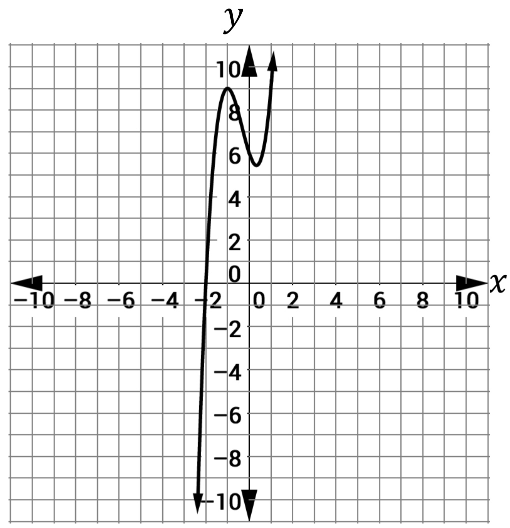
What are the other zeros of $h(x)$?

$x = -0.75$, and $x = 2.75$

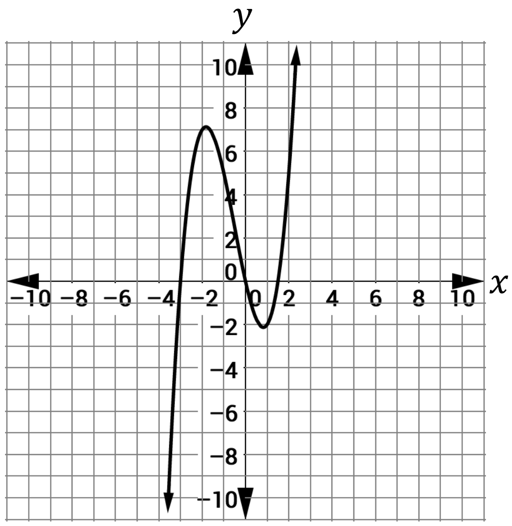
BEAT THE TEST!

1. Which of the graphs has the same zeros as the function $f(x) = 2x^3 + 3x^2 - 9x$?

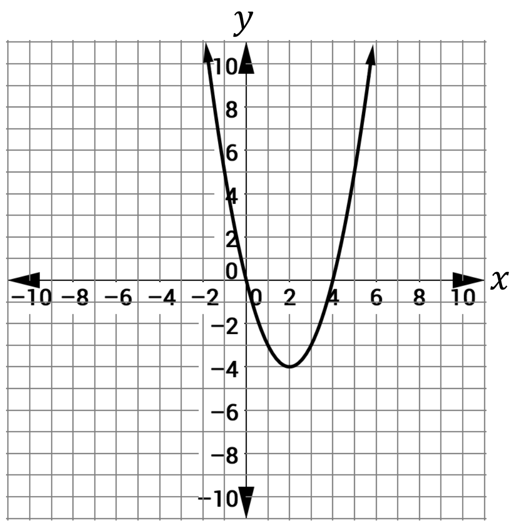
A



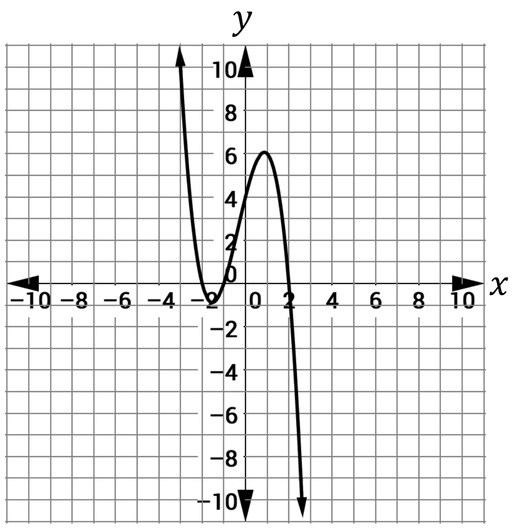
B



C



D



Answer: B

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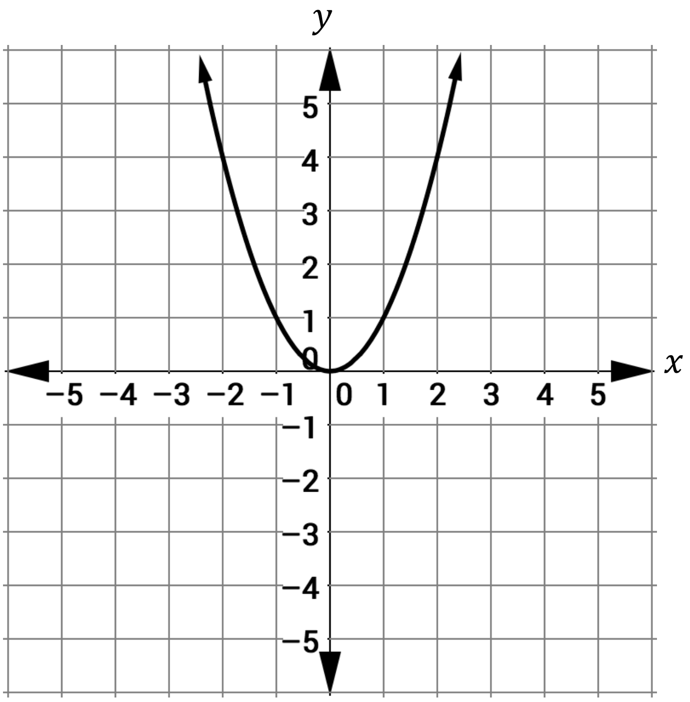
Section 8 – Topic 7
End Behavior of Graphs of Polynomials

Make observations about the end behavior of the following graphs.

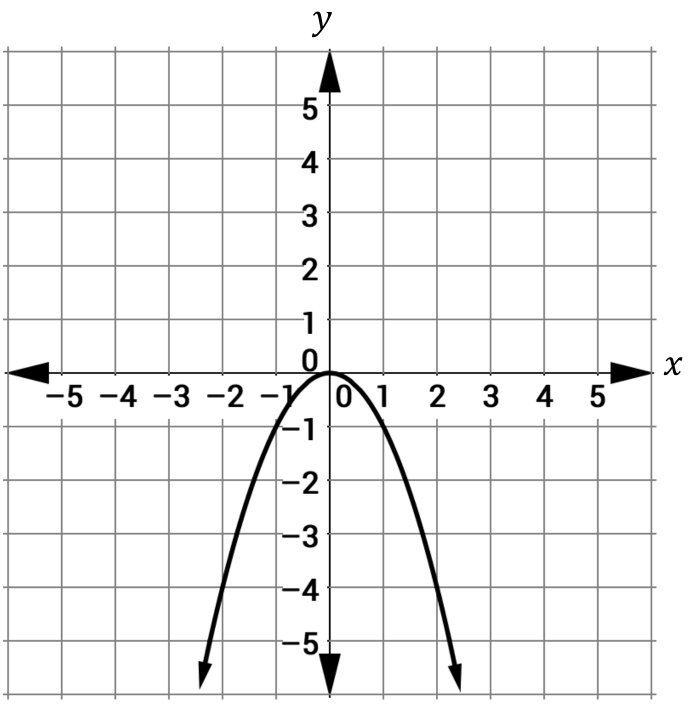
$y = x^2$

As x increases, y increases.
As x decreases, y increases.



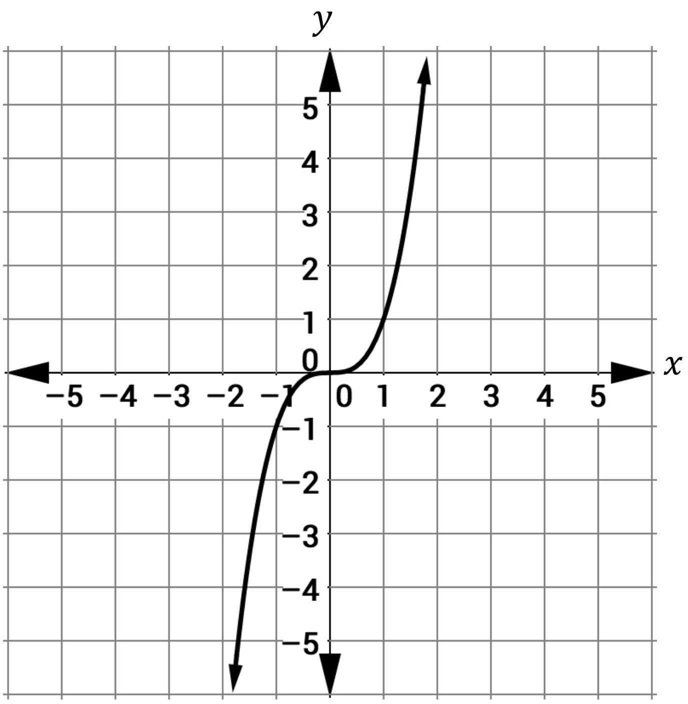
$y = -x^2$

As x increases, y decreases.
As x decreases, y decreases.



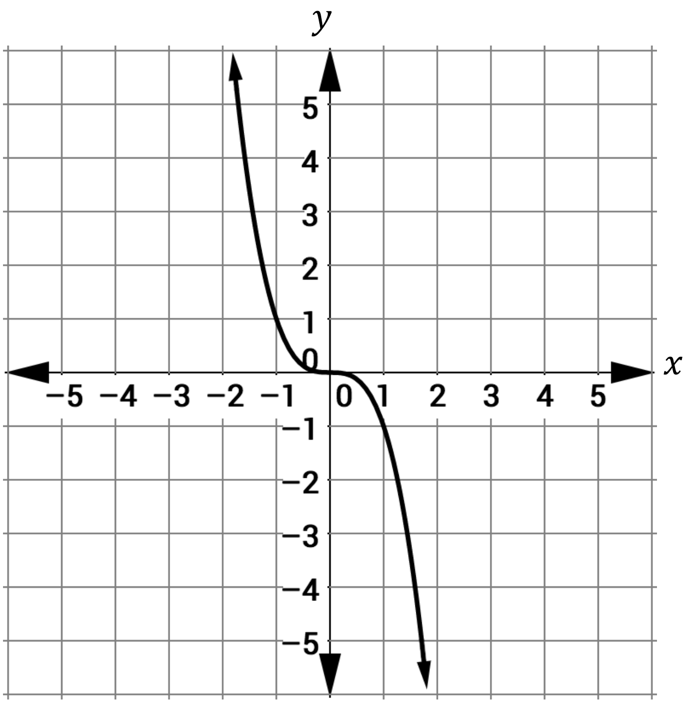
$y = x^3$

As x increases, y increases.
As x decreases, y decreases.



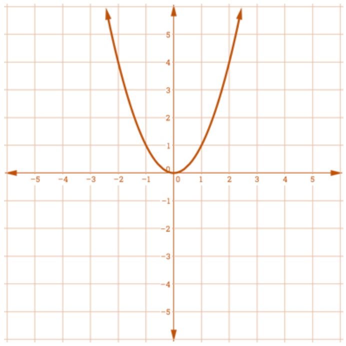
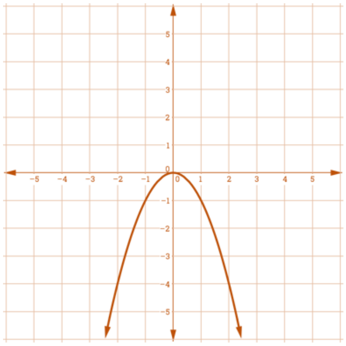
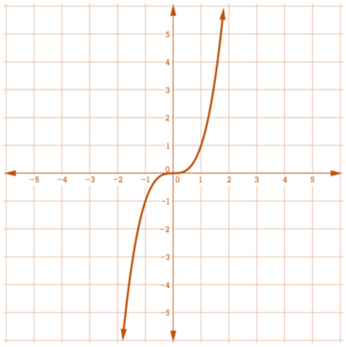
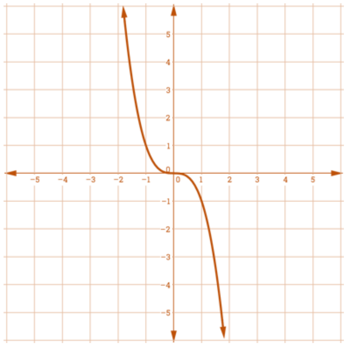
$y = -x^3$

As x increases, y decreases.
As x decreases, y increases.



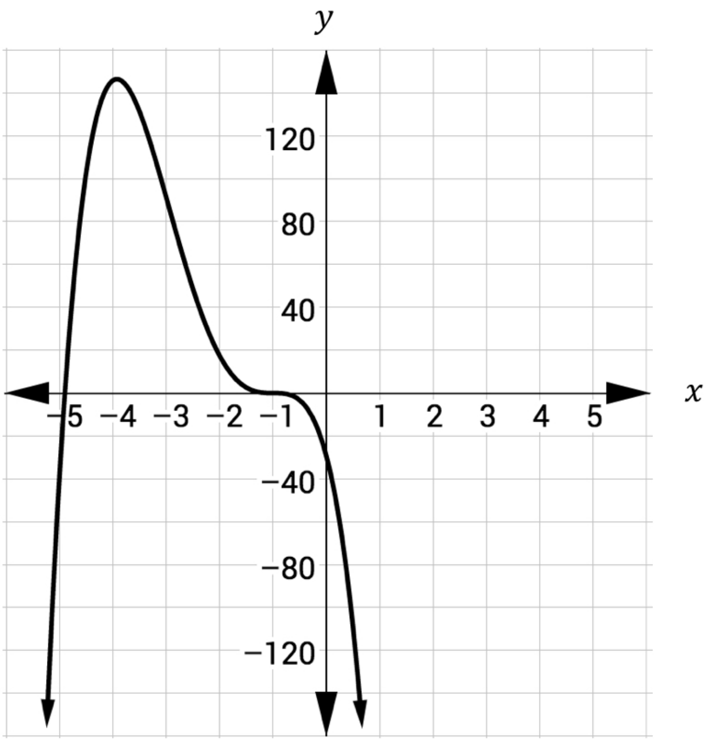
Use your observations to sketch the graphs and make conjectures to complete the table.

End Behavior of Polynomials

	Leading Coefficient is Positive	**Leading Coefficient is Negative**
Degree of Polynomial is Even	$f(x) = x^2$ 	$f(x) = -x^2$ 
	As $x \to \infty, f(x)\underline{\to \infty}$ As $x \to -\infty, f(x)\underline{\to \infty}$	As $x \to \infty, f(x)\underline{\to -\infty}$ As $x \to -\infty, f(x)\underline{\to -\infty}$
Degree of Polynomial is Odd	$f(x) = x^3$ 	$f(x) = -x^3$ 
	As $x \to \infty, f(x)\underline{\to \infty}$ As $x \to -\infty, f(x)\underline{\to -\infty}$	As $x \to \infty, f(x)\underline{\to -\infty}$ As $x \to -\infty, f(x)\underline{\to \infty}$

Let's Practice!

1. Consider the following graph of $f(x)$.



a. Does the function $f(x)$ have an even or odd degree? Justify your answer.

Even. The graph opens in the same direction on both sides.

b. Is the leading coefficient of $f(x)$ positive or negative? Justify your answer.

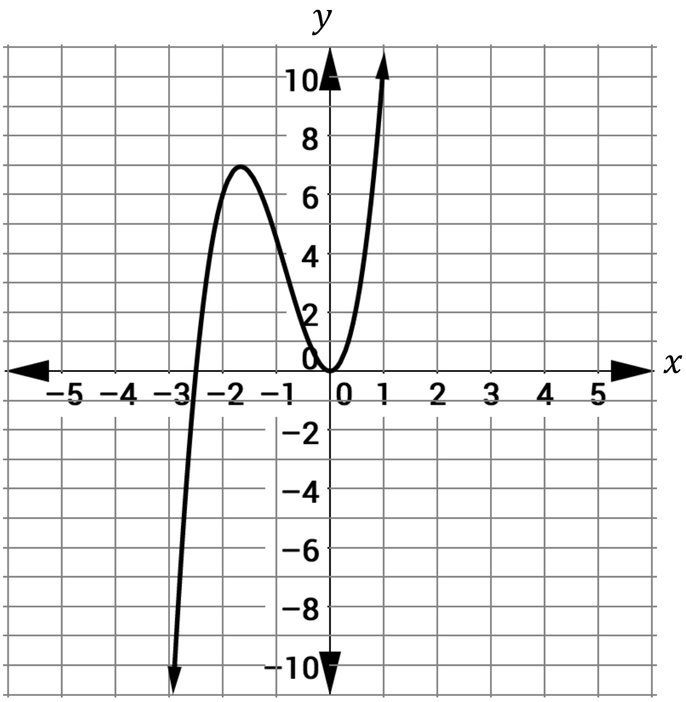
Negative. As $x \to \infty, f(x) \to -\infty$ and
As $x \to -\infty, f(x) \to -\infty$

2. Describe the end behavior of the function $g(x) = -5x^3 + 8x^2 - 9x$.

As $x \to \infty, f(x) \to -\infty$ and
As $x \to -\infty, f(x) \to \infty$

Try It!

3. Consider the following graph of $f(x)$.



a. Does the function $f(x)$ have an even or odd degree? Justify your answer.

Odd. The graph opens in opposite directions on the two sides.

b. Is the leading coefficient of $f(x)$ positive or negative? Justify your answer.

Positive. As $x \to \infty, f(x) \to \infty$ and
As $x \to -\infty, f(x) \to -\infty$

4. Describe the end behavior of the function below.

$$p(x) = \frac{1}{2}x^6 - x^5 - x^4 + 2x^3 - 2x + 2$$

As $x \to \infty, f(x) \to \infty$ and
As $x \to -\infty, f(x) \to \infty$

BEAT THE TEST!

1. Determine which of the following statements is true for the function $f(x) = 3x^5 + 7x - 4247$.

A As $x \to \infty, f(x) \to \infty$ and as $x \to -\infty, f(x) \to \infty$.
B As $x \to \infty, f(x) \to -\infty$ and as $x \to -\infty, f(x) \to -\infty$.
C As $x \to \infty, f(x) \to -\infty$ and as $x \to -\infty, f(x) \to \infty$.
D **As $x \to \infty, f(x) \to \infty$ and as $x \to -\infty, f(x) \to -\infty$.**

Answer is D.

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Section 8 – Video 8
Multiplicity of Roots in Repeated Factors

Consider the polynomial function $p(x) = x^5 - 2x^4 + x^3$

Determine the factors of $p(x)$.

$x^3(x^2 - 2x + 1)$
$x^3(x - 1)(x - 1)$

How many times does each factor appear in $p(x)$?

$(x - 1)$ twice
x three times

Determine the roots of $p(x)$.

$x = 1$
$x = 0$

The "multiplicity" of a root refers to the number of times the corresponding factor appears in a polynomial.

Determine the multiplicity of each root for $p(x)$.

$x = 0$ has a multiplicity of three
$x = 1$ has a multiplicity of two

Let's Practice!

Determine the root(s) and their multiplicity for the polynomial functions:

1. $f(x) = x^2 + 12x + 36.$

$(x + 6)^2$

$x = -6$ multiplicity of 2

2. $g(x) = (x^2 - 49)(x^2 + 8x + 7)(x^2 - 6x - 7)$

$(x + 7)(x - 7)(x + 7)(x + 1)(x - 7)(x + 1)$

$(x + 7)^2(x - 7)^2(x + 1)^2$

$x = 7$ with a multiplicity of two, $x = -1$ with a multiplicity of two, $x = -7$ with a multiplicity of two

Try It!

Determine the root(s) and their multiplicity for the polynomial functions:

3. $m(x) = x(x^2 + 4x - 5)(x + 5)$

$x(x + 5)(x - 1)(x + 5)$

$x(x + 5)^2(x - 1)$

$x = 0$ multiplicity of one, $x = 1$ multiplicity of one, $x = -5$ multiplicity of two

4. $r(x) = (x - 9)^3(x + 2)^4(x - 6)^2$

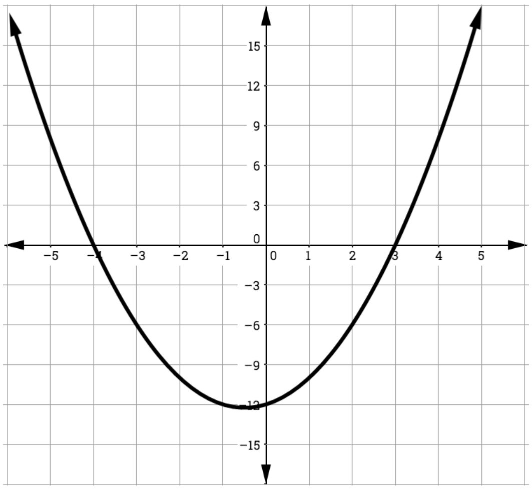
$x = 9$ multiplicity of three

$x = -2$ multiplicity of four

$x = 6$ multiplicity of two

Consider the following graphs. Determine the multiplicity of the roots of each function:

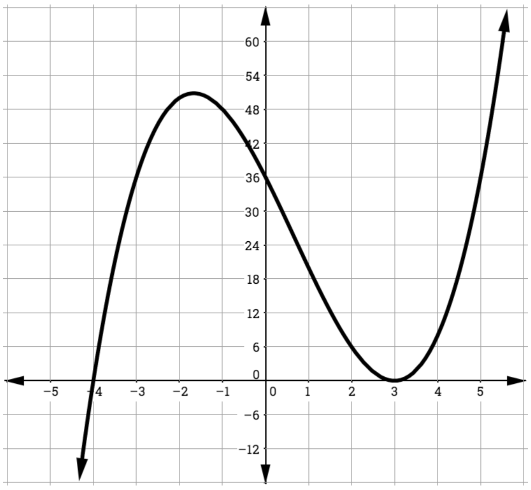
$b(x) = (x-3)(x+4)$



$x = -4$ __one______

$x = 3$ ___one______

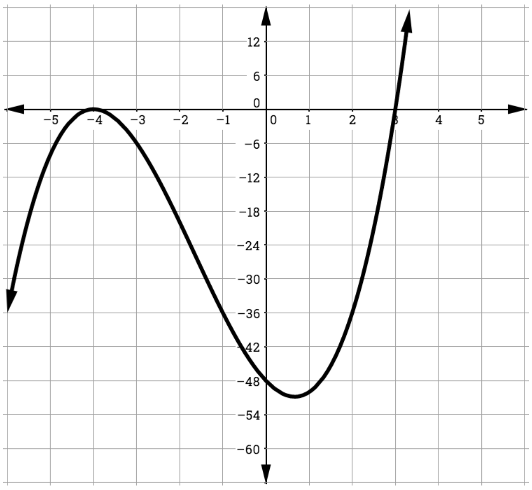
$c(x) = (x-3)^2(x+4)$



$x = -4$ __one______

$x = 3$ ___two______

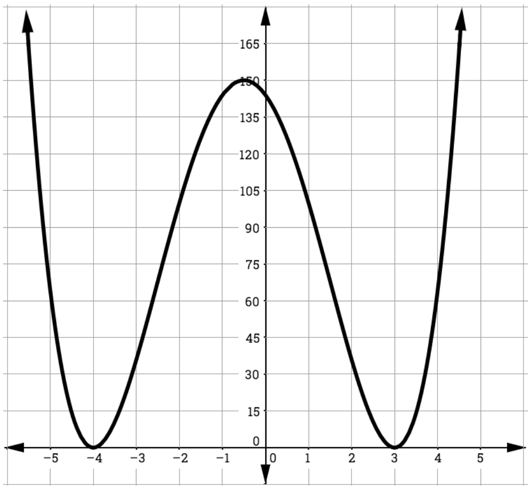
$d(x) = (x-3)(x+4)^2$



$x = -4$ ___two______

$x = 3$ ___one______

$e(x) = (x-3)^2(x+4)^2$



$x = -4$ __two______

$x = 3$ __two_______

What does the multiplicity of the zero tell us about the graph?

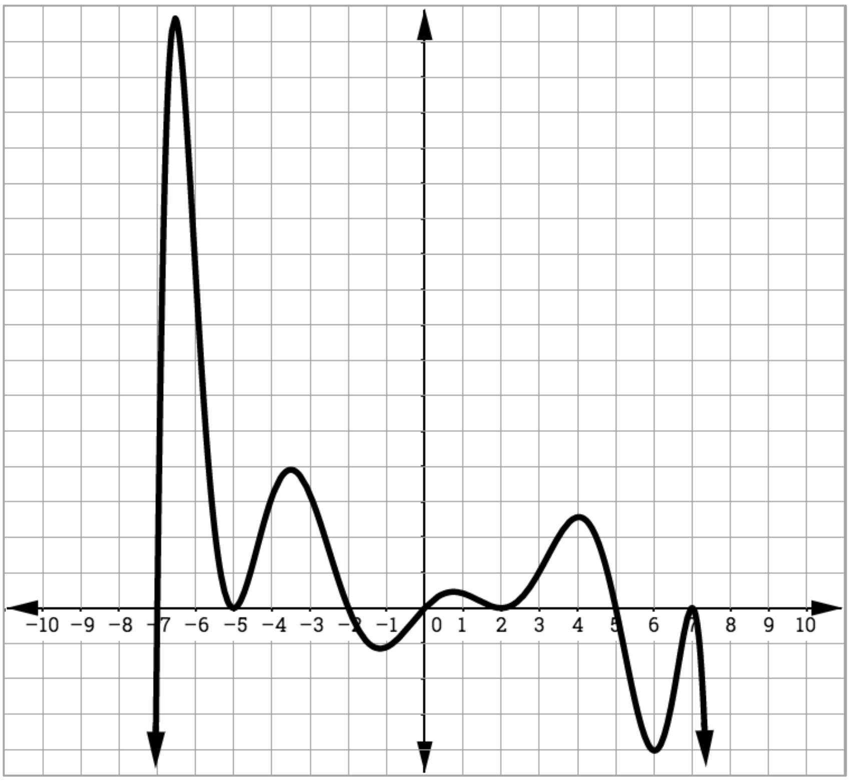
An odd multiplicity goes through the x-axis. An even multiplicity "bounce" off the x-axis.

Explain why the graph does not cross the x −axis when the multiplicity is an even number.

They are squares, so they don't change sign. Squares are always positive. This means that the x-intercept corresponding to an even-multiplicity zero can't cross the x-axis

Let's Practice!

5. The following graph shows a tenth-degree polynomial.

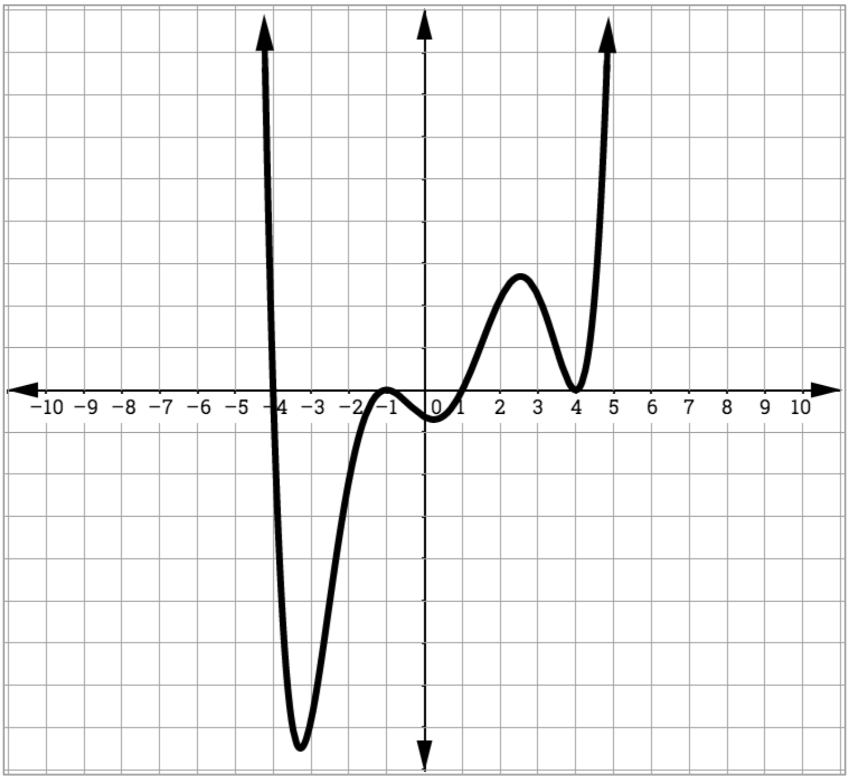


List the polynomial's zeros with their multiplicities.

$x = -7$ multiplicity of one, $x = -5$ multiplicity of two, $x = -2$ multiplicity of one, $x = 0$ multiplicity of one, $x = 2$ multiplicity of 2, $x = 5$ multiplicity of one, $x = 7$ multiplicity of two.

Try It!

6. The following graph shows a sixth-degree polynomial.

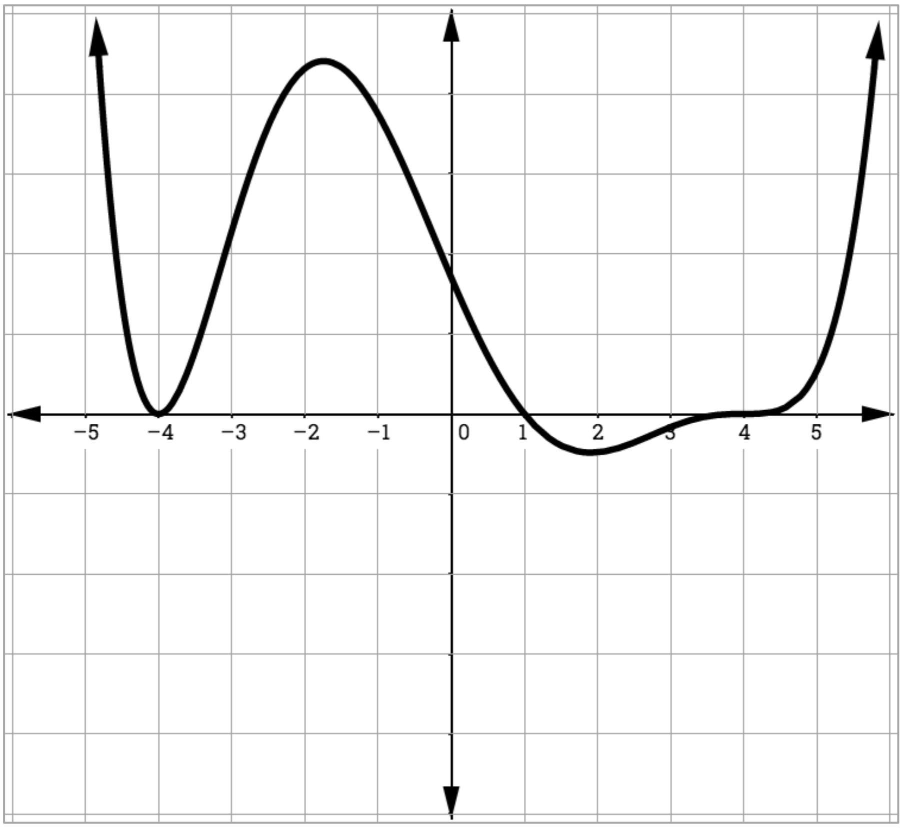


List the polynomial's zeros with their multiplicities.

$x = -4$ multiplicity of one, $x = -1$ multiplicity of two, $x = 1$ multiplicity of one, $x = 4$ multiplicity of two

BEAT THE TEST!

1. Consider the following graph.



Which of the following polynomials have the same zeros as the graph? Select all that apply.

☐ $f(x) = (x + 4)^2(x - 4)^3(x - 1)$
☐ $f(x) = (x - 1)(x + 4)(x - 4)^2$
☐ $f(x) = (x - 4)^2(x + 4)^2(x - 1)$
☐ $f(x) = (x + 4)^4(x - 1)^3(x - 4)^5$
☐ $f(x) = (x - 4)^2(x - 1)^4(x + 4)$
☐ $f(x) = (x - 1)^5(x - 4)^3(x + 4)$

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Section 8 – Topic 9
Graphing Polynomial Functions of Higher Degrees

Consider the following function.

$$g(x) = -(x+3)(x-1)(x-2)$$

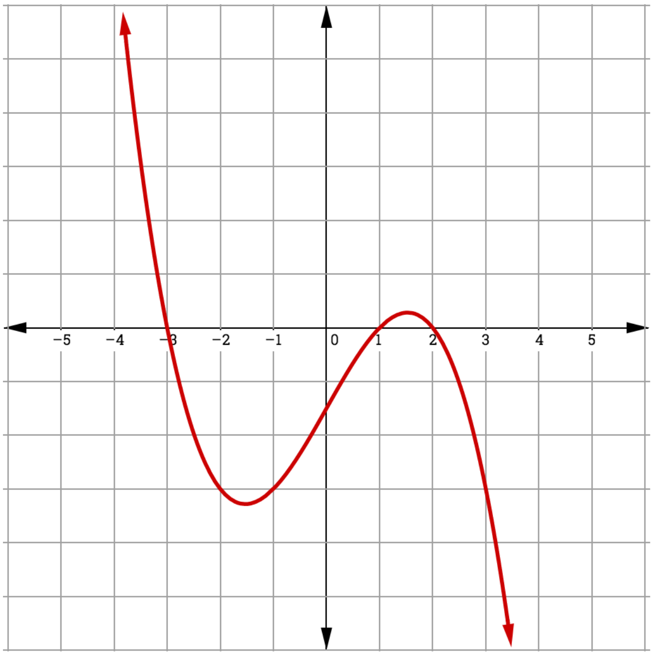
Describe the end behavior of the graph of $g(x)$.

As $x \to \infty$, $f(x) \to -\infty$ and As $x \to -\infty$, $f(x) \to \infty$

Find the zeroes of $g(x)$.

$x = -3$, $x = 1$, and $x = 2$

Use the end behavior and zeroes to sketch the graph of $g(x)$.



Let's Practice!

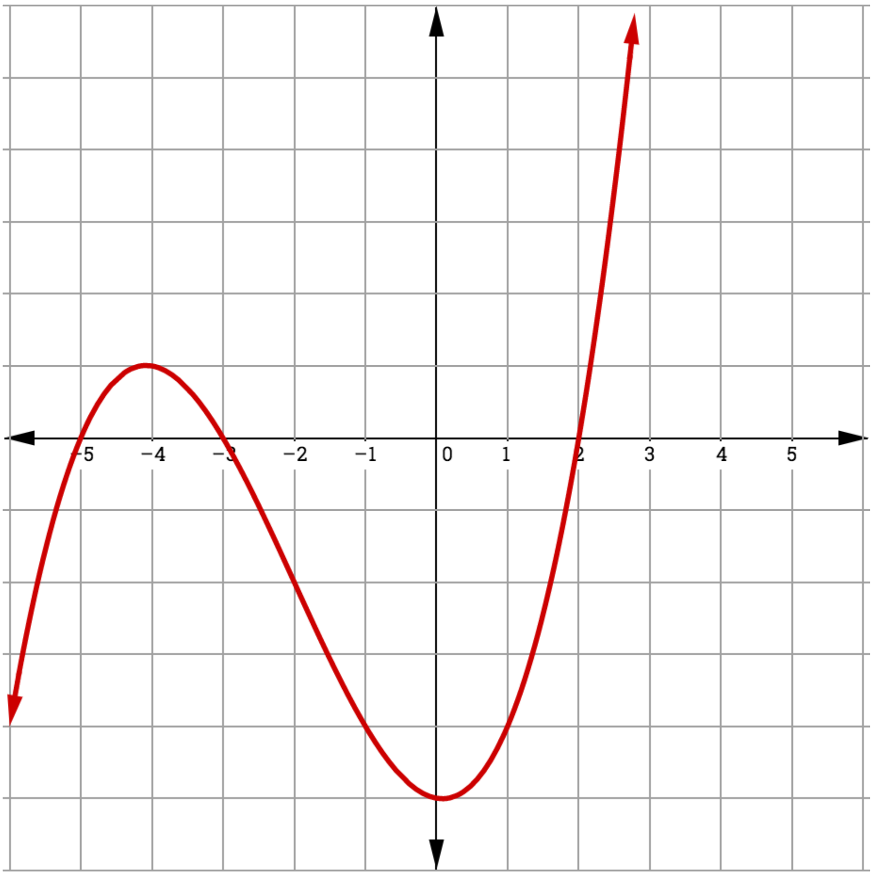
1. Sketch the graph of the following polynomial.

$$f(x) = (x-2)(x+3)(x+5)$$

End Behavior:

As $x \to \infty$, $f(x) \to \infty$ **and as** $x \to -\infty$, $f(x) \to -\infty$

Zeros: $x = 2$, $x = -3$, **and** $x = -5$



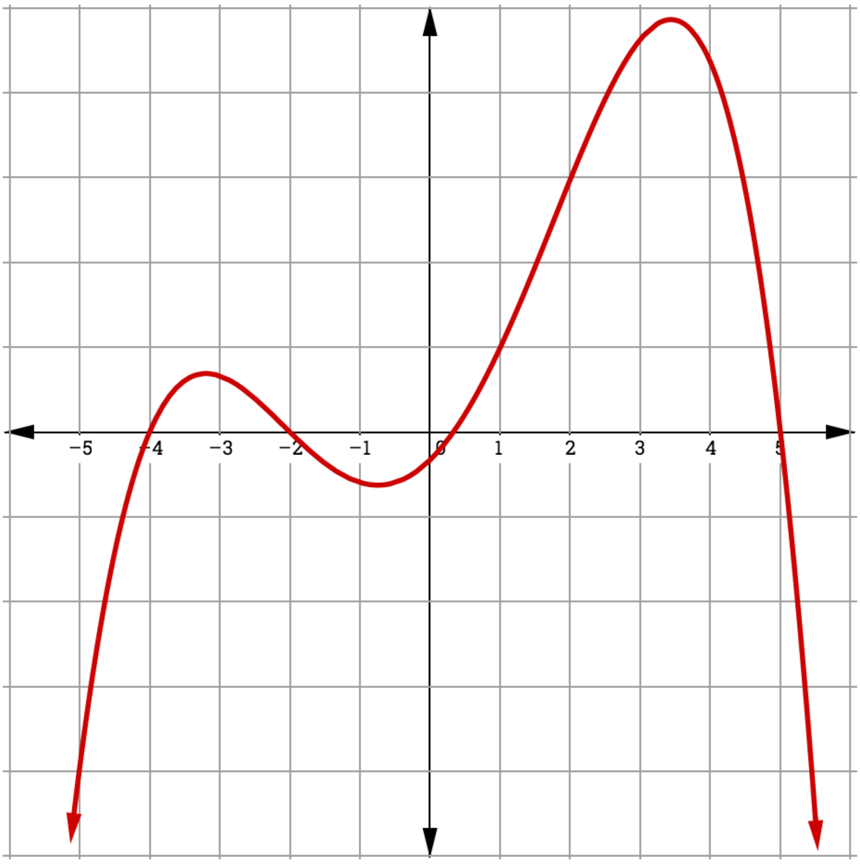
2. Sketch the graph of the following polynomial.

$$f(x) = -(x-5)(x+4)(3x-1)(x+2)$$

End Behavior:

As $x \to \infty$, $f(x) \to -\infty$ **and as** $x \to -\infty$, $f(x) \to -\infty$

Zeros: $x = 5$, $x = -4$, $x = \frac{1}{3}$, **and** $x = -2$



Try It!

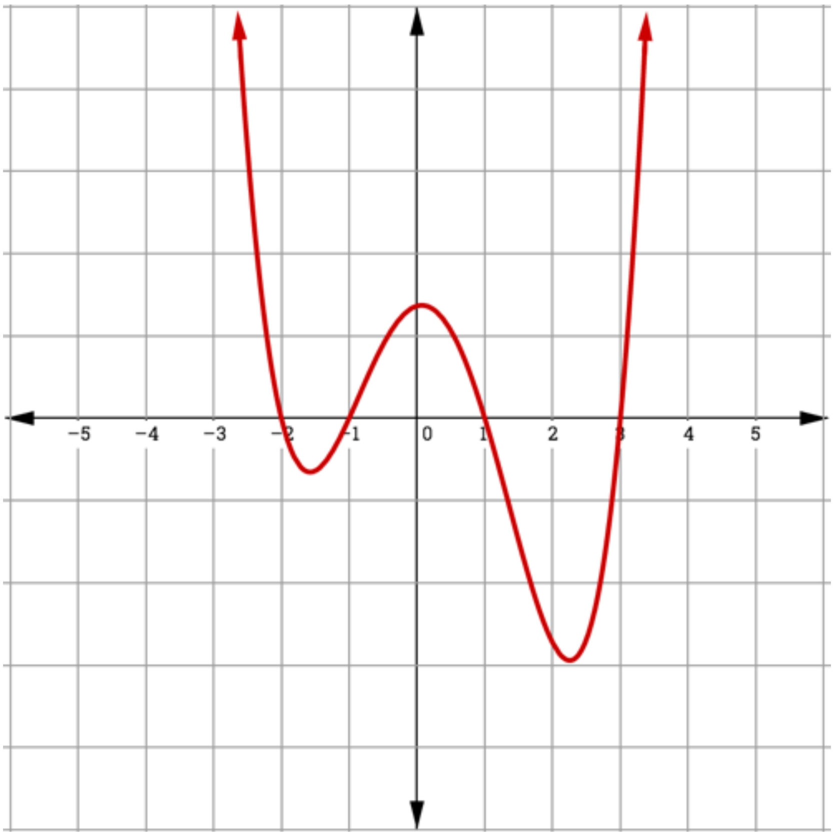
3. Sketch a graph of the following polynomial.

$$f(x) = (x-1)(x+2)(x-3)(x+1)$$

End Behavior:

As $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to \infty$

Zeros: $x = 1$, $x = -2$, $x = 3$, and $x = -1$

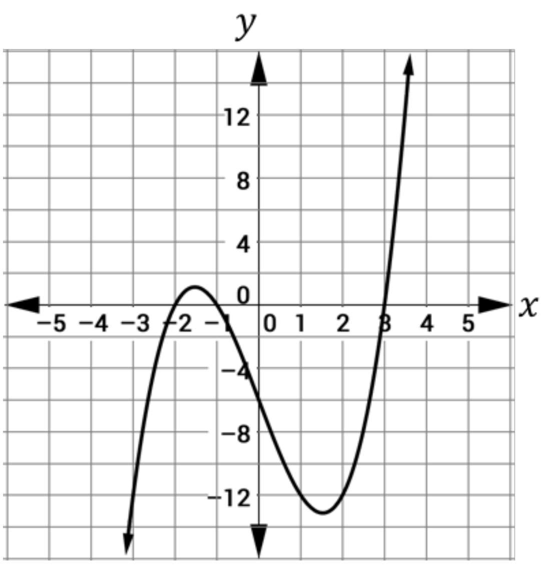


BEAT THE TEST!

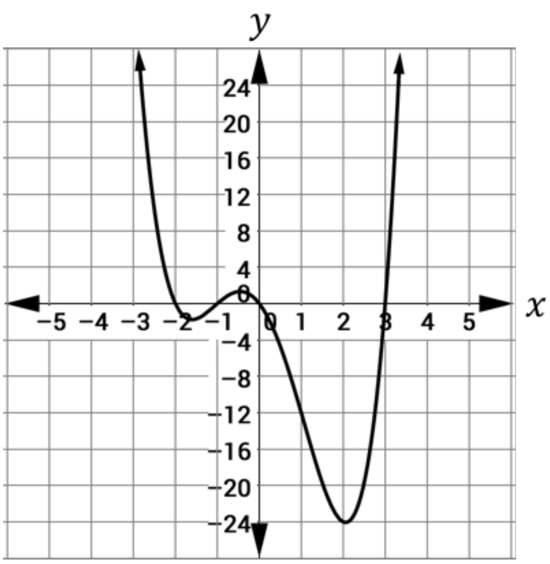
1. Match each equation with its corresponding graph.

A. $y = (x+1)(x-3)(x+2)$ **B.** $y = -(x+1)(x-3)(x+2)$

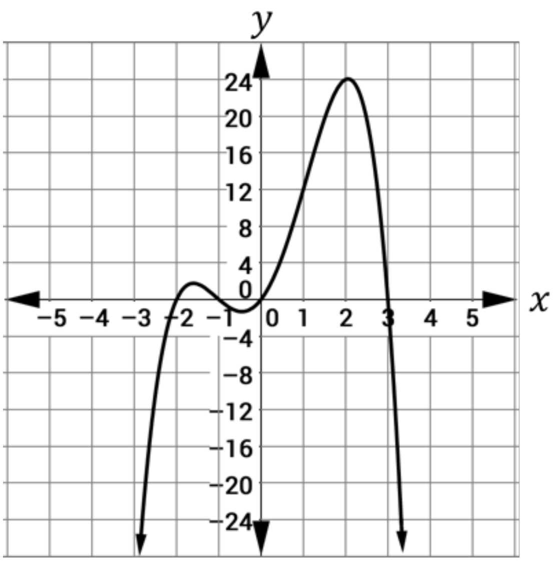
C. $y = -x(x+1)(x-3)(x+2)$ **D.** $y = x(x+1)(x-3)(x+2)$



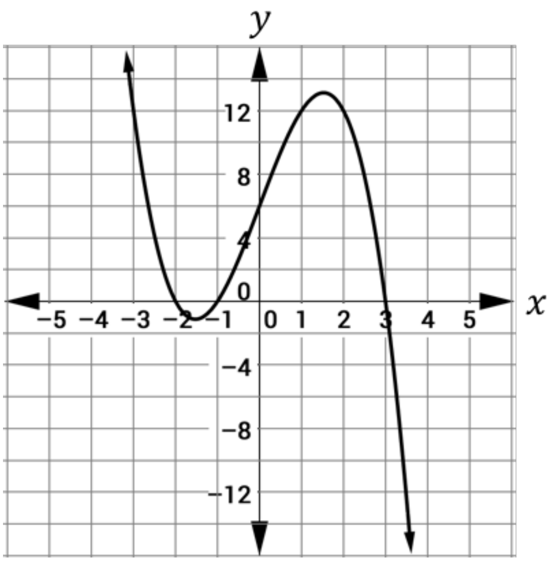
A



D



C



B

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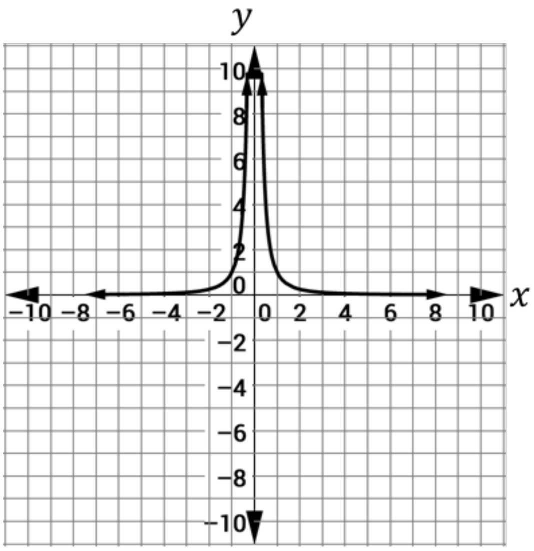
Section 8 – Topic 10
Recognizing Even and Odd Functions

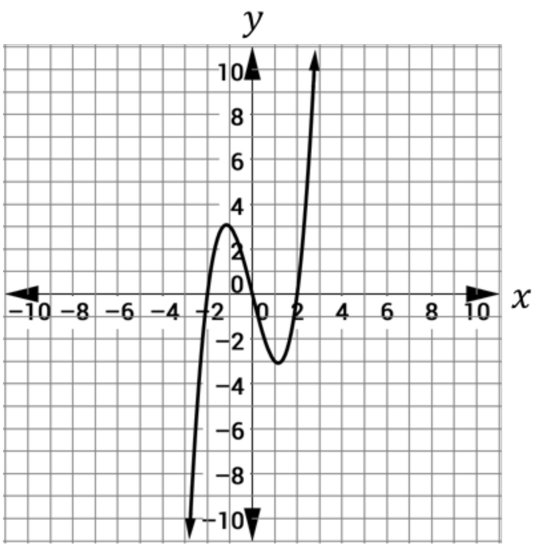
An even function has symmetry about the ___*y*-axis___.

An odd function has symmetry about the ___origin___.

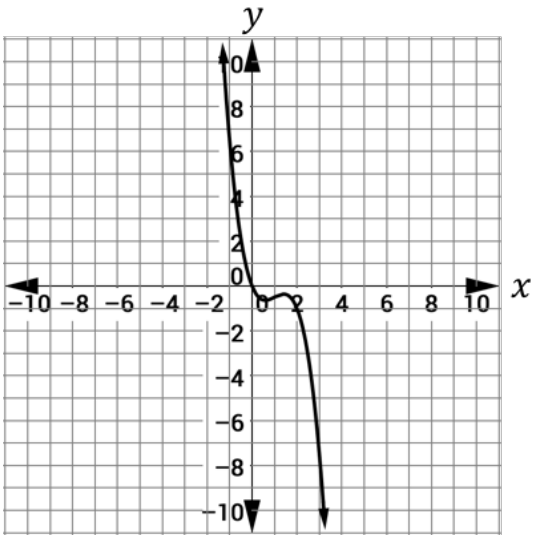
Consider the following graphs. Label each graph as even, odd, or neither in the space provided.



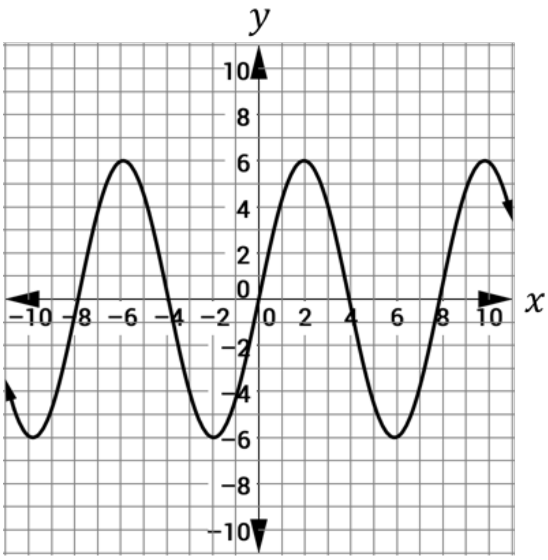
Even



Odd



Neither



Odd

If a function is even, then $f(-x) =$ $\underline{f(x)}$.

If a function is odd, then $f(-x) =$ $\underline{-f(x)}$.

To determine if a function is even or odd

- Substitute $(-x)$ into the function.
- If the resulting polynomial is the same, then the function is **even**.
- If the resulting polynomial is the exact opposite, then the function is **odd**.
- If the resulting polynomial is neither the same nor the exact opposite, the function is not even nor odd.

Let's Practice!

1. Complete the table below to determine if the following functions are even, odd, or neither.

Function	**Value of $f(-x)$**	**Even, Odd, or Neither?**
$f(x) = x^4 + x^3$	$f(-x) = (-x)^4 + (-x)^3$ $= x^4 - x^3$	**Neither**
$f(x) = x^6 + 1$	$f(-x) = (-x)^6 + 1 = x^6 + 1$	**Even**
$f(x) = 6x^5 - x^3$	$f(-x) = 6(-x)^5 - (-x)^3$ $= -6x^5 + x^3$	**Odd**

Try It!

2. Give an example of a polynomial function that is an even function.

$f(x) = x^6 + 1$
$f(-x) = x^6 + 1$

3. Give an example of a polynomial function that is an odd function.

$f(x) = x^3 + x$

4. Give an example of a polynomial function that is neither odd nor even.

$f(x) = x^3 + 1$ $\quad$ $f(-x) = -x^3 + 1$
$f(x) = x^4 + x^3$ $\quad$ $f(-x) = x^4 - x^3$

BEAT THE TEST!

1. Use the following functions to complete the table below.

$$f(x) = x^2$$
$$g(x) = x^3$$
$$h(x) = x^3 - 4x$$

Function	Even	Odd	Neither
$f(x) \cdot g(x)$	○	●	○
$g(x) \cdot h(x)$	●	○	○
$f(x) + h(x)$	○	○	●
$g(x) + h(x)$	○	●	○

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